Field Dependence of the Josephson Plasma Resonance in Layered Superconductors with Alternating Junctions

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(February 1, 2008)

The Josephson plasma resonance in layered superconductors with alternating critical current densities is investigated in a low perpendicular magnetic field. In the vortex solid phase the current densities and the squared bare plasma frequencies decrease linearly with the magnetic field. Taking into account the coupling due to charge fluctuations on the layers, we extract from recent optical data for $SmLa_{1-x}Sr_xCuO_{4-\delta}$ the Josephson penetration length $\lambda_{ab} \approx 1100$ Å parallel to the layers at $T = 10$ K.

PACS numbers: 74.25.Gz, 42.25.Gy, 74.72.-h, 74.80.Dm

The suppression of the Josephson interlayer coupling and consequently the Josephson plasma resonance (JPR) frequency, ω_0, due to misaligned pancake vortices in layered superconductors with identical intrinsic junctions was calculated in the Refs. [1–4] in good agreement with experimental data in the $Bi_2Sr_2CaCu_2O_{8-\delta}$ superconductor. It was found that in the vortex crystal phase, at low fields, ω_0 drops linearly with the vortex concentration (with the magnetic field B applied along the c-axis), while in the vortex liquid and glass state, at high fields, ω_0 drops as $1/\sqrt{B}$ because vortices are uncorrelated along the c-axis and many of them contribute to the suppression of the Josephson coupling at a given point.

Recently the JPR was studied by optical means [5,6] in the layered superconductor $SmLa_{1-x}Sr_xCuO_{4-\delta}$ with alternating intrinsic Josephson junctions containing SmO and LaO as insulating barriers between superconducting CuO_2-layers. Reflectivity and transmission measurements were performed with the incidence of light parallel to the layers (ab-plane) and the polarization of the electric field in c-direction. Then the effective dielectric function $\epsilon_{\rm eff}(\omega)$ was extracted from the data by using the Fresnel formulae. In crystals with $x = 0.2$ two peaks in the loss function, $L(\omega) = {\rm Im}[-1/\epsilon_{\rm eff}(\omega)]$, were found at frequencies 6.6 and 8.9 cm^{-1} at $B = 0$. These frequencies drop with B linearly at low $B < 0.5$ T and as $1/\sqrt{B}$ at $B > 1$ T. The behavior of the JPR frequencies at high magnetic fields was attributed to the vortex liquid state.

The relative intensities of these peaks and the dependence of the JPR frequencies on B at high fields were explained in the Refs. [10–12] in the model of two alternating intrinsic junctions with different Josephson critical current densities. Thereby it is essential to take into account the charge coupling of neighboring junctions, i.e. the c-axis spatial dispersion of Josephson plasmons [7–9], in order to explain the peak amplitudes both for parallel and grazing incidence on the surface [10–13]. The dimensionless parameter α characterizing the change of the chemical potential of superconducting layers with the electron concentration was estimated from the field dependence of the JPR resonances in the loss functions [11].

In this paper we will calculate the behavior of the JPR frequencies at low magnetic fields. This allows us to extract another important parameter of the $SmLa_{1-x}Sr_xCuO_{4-\delta}$ superconductor, the London penetration length λ_{ab}.

We consider a crystal with alternating Josephson critical current densities J_m, $m = 1, 2$, and corresponding bare plasma frequencies $\omega_{0m}^2 = 8\pi^2 csJ_m/\epsilon_0\Phi_0$, where s is the interlayer spacing which we assume to be similar in both junctions (for $SmLa_{1-x}Sr_xCuO_{4-\delta}$ the difference is about 1.5%) and ϵ_0 is the high frequency dielectric constant (≈ 19 for $SmLa_{1-x}Sr_xCuO_{4-\delta}$, see Ref. [11]). We calculate first the dependence of the bare JPR frequencies $\omega_{0m}(B)$ on B without charge coupling between the junctions. Then we will account for the JPR dispersion due to α to find the real JPR eigenfrequencies ω_m and the field dependence of the resonances ω_m in the loss function $L(\omega)$.

Let us consider the field dependence of the JPR frequencies in the single vortex regime at low fields $B \ll B_{Jm}, B_\lambda$, where $B_J = \Phi_0/\lambda_{Jm}^2$, $B_\lambda = \Phi_0/4\pi\lambda_{ab}^2$ and $\lambda_{Jm} = (\Phi_0 cs/8\pi^2\lambda_{ab}^2 J_m)^{1/2}$ are the Josephson lengths of the junctions of type $m = 1, 2$. In this limit the interaction between the vortices is screened (single vortex regime). We neglect pinning assuming that mainly thermal fluctuations are responsible for the meandering of the vortex line. This assumption is valid at sufficiently high temperatures T. In Ref. [3] it was shown that in the single vortex regime the field dependence is given as

$$\omega_{0m}^2(B) = \omega_{0m}^2(0)(1 - B/B_{0m}), \quad B_{0m} = \Phi_0/I_m, \qquad (1)$$

$$I_m = \int d\mathbf{r}\langle[1 - \cos\varphi_{n,1;n+1-m,2}(\mathbf{r})]\rangle, \qquad (2)$$

where $\mathbf{r}$ is the in-plane coordinate, $\varphi_{n,1;n-m+1,2}(\mathbf{r})$ is the phase difference between the layers $n1$ and $n - m + 1, 2$ and $\langle \ldots \rangle$ is the average over thermal disorder. We introduce the displacements u_{nm} of the vortex from the

straight line along the c-axis in the layers nm and its Fourier transforms $u_m(q)$ with respect to the coordinate n of the unit cell consisting of two different junctions, $0 \leq q \leq 2\pi$. Then we express the phase difference via vortex displacements in quadratic approximation as it was done in Ref. [3] and obtain

$$I_1 = \frac{\pi}{2}\left\langle \int \frac{dq}{2\pi}(1-\cos q)|u_1(q)-u_2(q)|^2 \times \ln\left(\frac{3.72\lambda_{J1}^2}{(u_{n1}-u_{n2})^2(1-\cos q)}\right)\right\rangle, \quad (3)$$

$$I_2 = \frac{\pi}{2}\left\langle \int \frac{dq}{2\pi}(1-\cos q)|u_1(q)-u_2(q)e^{iq}|^2 \times \ln\left(\frac{3.72\lambda_{J2}^2}{(u_{n1}-u_{n-1,2})^2(1-\cos q)}\right)\right\rangle. \quad (4)$$

In the following we will use the self-consistent harmonic approximation (SCHA) replacing $(u_{n1}-u_{n2})^2$ by its average value $\langle(u_{n1}-u_{n2})^2\rangle = 4r_{w1}^2$, where r_{wm} is the meandering length for junctions of the type m. I_m in terms of r_{wm} is

$$I_m = (\pi/2)r_{wm}^2 \ln(0.8\lambda_{Jm}/r_{wm}). \quad (5)$$

The increase of the vortex energy due to displacements is

$$\mathcal{E}_{\rm vor} = \frac{1}{2}\sum_q [E_{J1}|u_1(q)-u_2(q)|^2 + E_{J2}|u_1(q)-u_2(q)e^{iq}|^2 + W_M(|u_1(q)|^2+|u_2(q)|^2)], \quad (6)$$

where $E_{Jm} = \Phi_0 J_m/2c$ is the Josephson coupling density in a junction of the type m multiplied by the factor $(\pi/2)$ and $W_M = \Phi_0^2 s/(4\pi\lambda_{ab}^2)^2$ accounts for the cage potential due to the nonlocal magnetic interaction between pancakes in different layers. After diagonalization of this energy we obtain the free energy functional of the vortex line

$$\mathcal{F} = \frac{1}{2}\sum_{q,i=1,2}[E_i(q)|v_i(q)|^2 - 2T\ln|v_i(q)|^2], \quad (7)$$

$$E_{1,2}(q) = E_{J1}+E_{J2}+W_M \pm (E_{J1}^2+E_{J2}^2+2E_{J1}E_{J2}\cos q)^{1/2}. \quad (8)$$

After minimization with respect to the new variables $v_i(q)$ we obtain the free energy as

$$F(E_{Jm},W_M,T) = 2T\sum_q \ln[E_1(q)E_2(q)] + F_0(T), \quad (9)$$

For I_m we derive (neglecting logarithmic factors)

$$I_1 = \frac{\pi r_{w1}^2}{2} = \frac{\pi}{2}\frac{\partial F}{\partial E_{J1}} \approx \frac{\pi T}{E_{J1}} f(E_{J1}),$$
$$f(E_{J1}) = \left[1 - \frac{1+2\mathcal{E}_{J2}}{\sqrt{(1+4\overline{\mathcal{E}}+4\mathcal{E}_{J1}\mathcal{E}_{J2})(1+4\overline{\mathcal{E}})}}\right], \quad (10)$$

and similar for I_2, where $\mathcal{E}_{Jm} = E_{Jm}/W_M = (\lambda_{ab}/\lambda_{Jm})^2 = (\lambda_{ab}^2/\lambda_{cm}s)^2$ and $\overline{\mathcal{E}} = (\mathcal{E}_{J1}+\mathcal{E}_{J2})/2$. The values λ_{cm} are determined by the zero field bare plasma frequencies $\omega_{0m}(B=0) = c/\sqrt{\epsilon_0}\lambda_{cm}$ in the absence of an external magnetic field B.

For high fields $B \gg B_{Jm}, B_\lambda$ the vortex lattice is more dense, the interaction between the vortices diminishes pancake fluctuations, while the enhanced tilt stiffness of the lattice favours larger fluctuations, i.e. a reduced Josephson energy. We write down the energy functional in the Fourier representation $\mathbf{k}$ with respect to the in-plane coordinate $\mathbf{r}$ and the unit cell index n. We separate transverse, $u_t(\mathbf{k},n)$, and longitudinal, $u_l(\mathbf{k},n)$, vortex lattice displacements (for details see Ref. [3])

$$\mathcal{E}_{\rm vor} = \mathcal{E}_t(u_t) + \mathcal{E}_l(u_l), \quad (11)$$

$$\mathcal{E}_t(u_t) = \sum_{\mathbf{k},n,m}[C_{66}k^2 + \Phi_{44}(\mathbf{k},m)]|u_t(\mathbf{k},2n+m)|^2, \quad (12)$$

$$\mathcal{E}_l(u_l) = \sum_{\mathbf{k},n,m}[\Phi_{11} + \Phi_{44}(\mathbf{k},m)]|u_l(\mathbf{k},2n+m)|^2, \quad (13)$$

where the summation over $\mathbf{k}$ is limited to the first Brillouin zone, which we approximate by the circle $k < K_0$, $K_0^2 = 4\pi B/\Phi_0$. Further, $\Phi_{11} \approx (B^2 s/4\pi\lambda_{ab}^2)(1-k^2/4K_0^2)$ is the compression stiffness, $C_{66} = A_{66}B\Phi_0 s/(8\pi\lambda_{ab})^2$ is the shear modulus. The parameter $A_{66} < 1$ describes the fluctuation suppression of C_{66}, cf. Ref. [14], which we approximate as $A_{66} = 1 - 0.4B/B_{\rm melt}$ with $B_{\rm melt}$ being the melting field, and

$$\Phi_{44}(k,m) = E_{cm} + E_{Jm}\eta_m \quad (14)$$

is the tilt stiffness. The cage energy is given as

$$E_{cm} = \frac{B\Phi_0 s}{2(4\pi\lambda_{ab}^2)^2}\ln\left(0.5 + \frac{0.13a^2}{r_{wm}^2}\right). \quad (15)$$

Here $a^2 = \Phi_0/B$ is the intervortex distance and

$$\eta_m = \frac{B}{2\Phi_0}\ln\frac{0.11a^2}{r_{wm}^2(1-0.53k^2/K_0^2)^2} + \frac{4\pi}{a^4k^2}. \quad (16)$$

After diagonalization of the energy functional we obtain the eigenvalues E_{t1} and E_{t2} for the transverse displacements with

$$E_{t1}E_{t2} = S_{t1}S_{t2} + (S_{t1}+S_{t2})(E_{J1}\eta_1 + E_{J2}\eta_2) + 2E_{J1}E_{J2}\eta_1\eta_2(1-\cos q), \quad (17)$$

$$S_{tm} = C_{66}k^2 + E_{cm}. \quad (18)$$

For longitudinal eigenvalues E_{lm} we need to replace S_{tm} with $S_{lm} = \Phi_{11} + E_{cm}$.

Finally we find the free energy and by differentiating it with respect to E_{Jm} we find the self-consistency equations for r_{wm} as

$$I_m = \pi r_{wm}^2/2 = \pi(r_{wt,m}^2 + r_{wl,m}^2)/2, \quad (19)$$

$$r_{wt,1}^2 = \frac{\Phi_0}{B} T \int \frac{d\mathbf{k}dq}{(2\pi)^3} \frac{[2E_{J2}\eta_2(1-\cos q) + S_{t1} + S_{t2}]\eta_1}{E_{t1}E_{t2}}$$

$$= \frac{\Phi_0}{B} \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{T}{E_{J1}} \left[1 - \frac{S_{t1}S_{t2} + (S_{t1}+S_{t2})E_{J2}\eta_2}{\sqrt{D_t^2 - (2E_{J1}E_{J2}\eta_1\eta_2)^2}}\right],$$

$$D_t = S_{t1}S_{t2} + (S_{t1}+S_{t2})(E_{J1}\eta_1 + E_{J2}\eta_2) + 2E_{J1}E_{J2}\eta_1\eta_2,$$

and similar for $r_{wt,2}^2$ and $r_{wl,m}^2$. The Eqs. (16)-(20) should be solved self-consistently to find the meandering lengths r_{wm}.

The bare frequencies $\omega_{0m}(B)$ determine the resonance frequencies $\omega_m(B)$, which are renormalized due to the charge coupling of the layers, and are observed experimentally in reflectivity, transmissivity and in the loss function $L(\omega)$. Next we derive the renormalized frequencies and compare them with the measurement of these quantities in Ref. [6]. The parameter $\alpha = (\epsilon_0/4\pi es)(\partial\mu/\partial\rho)$ characterizes the interlayer coupling due to charge fluctuations [7–9], where μ and ρ are the chemical potential and charge density on the layers, respectively. The parameter α was estimated as 0.4 for $SmLa_{1-x}Sr_xCuO_{4-\delta}$ at $x = 0.2$ in Ref. [11] both from the magnetic field dependence of the JPR resonances in the vortex liquid state and from the relative amplitude of the resonances in $L(\omega)$ for $B = 0$. For incident light perpendicular to the crystal surface ac the wave vector, k_x, of the wave propagating into the crystal determines the reflection coefficient, $R(\omega)$, according to the Fresnel formula, $R = |(1-n)/(1+n)|^2$, with the refraction index $n(\omega) = ck_x(\omega)/\omega$. In Ref. [11,12] the dispersion relation $k_x(\omega)$ was calculated using the Maxwell equations and the equation for the phase differences in a stack of intrinsic Josephson junctions coupled inductively and due to charge variations inside the layers. The result is

$$\frac{c^2k_x^2}{\omega^2\epsilon_0} = \frac{\epsilon_{\rm eff}(w)}{\epsilon_0} = \frac{r(w-v_1)(w-v_2)+iS}{rw^2 - (1+r)(2\alpha+1/2)w + iS_1}, \quad (20)$$

$$v_m = \frac{\omega_m^2}{\omega_{01}^2}, \quad v_{1,2} = (1+r)(1+2\alpha)\frac{1 \mp \sqrt{1-p}}{2r}, \quad (21)$$

$$r(B) = \frac{\omega_{01}^2}{\omega_{02}^2} < 1, \quad p = \frac{4r(1+4\alpha)}{(1+r)^2(1+2\alpha)^2}, \quad (22)$$

$$S_1 = w^{3/2} r(2\alpha+1/2)(\tilde\sigma_1 + \tilde\sigma_2), \quad (23)$$

$$S = w^{1/2}[(2\alpha+1)rw(\tilde\sigma_1+\tilde\sigma_2) - (1+4\alpha)(\tilde\sigma_1 + \tilde\sigma_2 r)],$$

where $\omega_m(B,\alpha)$ are the normalized JPR frequencies, $w = \omega^2/\omega_{01}^2$, $\tilde\sigma_m = 4\pi\sigma_m/\epsilon_0\omega_{01}$, and σ_m are the c-axis quasiparticle conductivities of the junctions. We see that the loss function $L(\omega)$ has two peaks at $\omega = \omega_m$ corresponding to zeros of $\epsilon_{\rm eff}(\omega)$ in the absence of dissipation. These resonances in the loss function correspond to the transverse plasma modes propagating along the layers. There is another characteristic frequency defined by the relation

$$\omega_{\rm pole}^2(B) = [\omega_{01}^2(B) + \omega_{02}^2(B)](2\alpha+1/2), \quad (24)$$

which corresponds to a pole of $\mathrm{Im}(\epsilon_{\rm eff}(\omega))$ in the absence of dissipation, cf. Eq.(20). This frequency is near the minimum of the loss function $L(\omega)$. This frequency is often called "transverse plasmon" and it is well defined experimentally as the peak in the real part of the optical conductivity, $\mathrm{Im}(\omega\epsilon_{\rm eff})$, see Ref. [6].

Next we calculate the field dependence of the frequency $\omega_{\rm pole}$, expressed by Eq. (24) via the field dependence of the bare frequencies ω_{0m}, Eq. (1), which we already found. We obtain

$$\omega_{\rm pole}^2(B) = \omega_{\rm pole}^2(0)(1 - B/B_p). \quad (25)$$

In the single vortex regime ($B \ll B_{Jm}, B_\lambda$) the result is

$$B_p = \frac{\Phi_0^3\epsilon_0(\omega_{01}^2(0) + \omega_{02}^2(0))}{32\pi^3c^2sTg(\mathcal{E}_{J1},\mathcal{E}_{J2})}, \quad (26)$$

$$g(\mathcal{E}_{J1},\mathcal{E}_{J2}) = (1/2)[f(E_{J1}) + f(E_{J2})] \quad (27)$$

$$= 1 - \frac{1+\overline{\mathcal{E}}}{\sqrt{(1+4\overline{\mathcal{E}} + 4\mathcal{E}_{J1}\mathcal{E}_{J2})(1+4\overline{\mathcal{E}})}}. \quad (28)$$

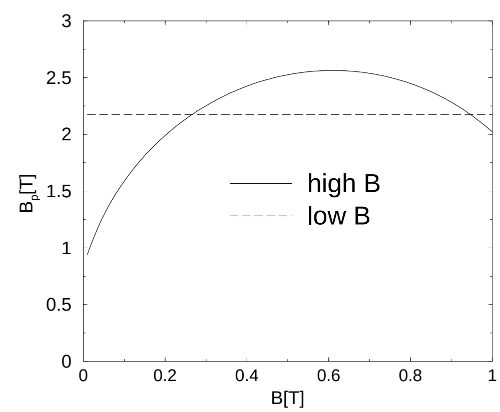

FIG. 1. Field scale $B_p(B)$ in the low ($B \ll B_{Jm}, B_\lambda$) or high ($B \gg B_{Jm}, B_\lambda$) field limit using Eq. (10) (dashed) or Eq. (19) (solid, $B_{\rm melt} = 2$ T) for $\lambda_{ab} = 1700$ Å, $T = 2.3$ K.

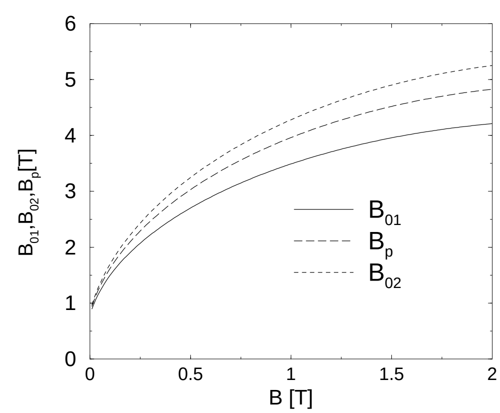

FIG. 2. Field scales B_{0m} and B_p for the variation of the JPR resonances ω_m and the pole $\omega_{\rm pole}$ in the limit of high fields $B \gg B_\lambda$, cf. Eq. (19) ($B_{\rm melt} = 2$ T, $\lambda_{ab} = 1700$ Å, $T = 2.3$ K).

In Fig. 1 it is seen that in the low field limit $B \ll B_{Jm}$ as determined in Eq. (10) the field scale B_p determining the variation of the peak frequency $\omega_{\rm pole}$ in $\mathrm{Im}(\omega\epsilon_{\rm eff})$ is field independent. In contrast to this, the self-consistent solution of Eq. (19) valid for $B \gg B_\lambda$ introduces a variation of B_p with B. In Fig. 2 the field scales B_{0m} and B_p responsible for the variation of the JPR peaks ω_m in $L(\omega)$ and of the pole $\omega_{\rm pole}$ in $\mathrm{Im}(\omega\epsilon_{\rm eff})$ are compared. The sensitivity of the magnetic field scale B_{0m} to the choice of the correct melting field is low.

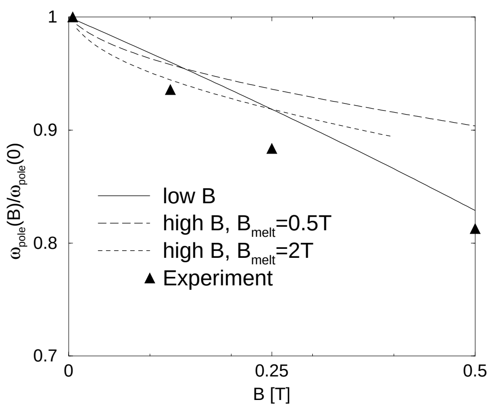

FIG. 3. Frequency $\omega_{\rm pole}/\omega_{\rm pole}(0)$ of the peak in $\mathrm{Im}(\omega\epsilon_{\rm eff})$ in the low and high field limit $B \ll B_{Jm}, B_\lambda$ or $B \gg B_{Jm}, B_\lambda$ using Eq. (10) or Eq. (19) respectively ($T = 2.3$ K). Best fit of the experimental data in Ref. [6] is generally obtained for the penetration depth $\lambda_{ab} = 1850$ Å, except for the case, when a too large melting field $B_{\rm melt} = 2$ T is chosen for comparison, which requires $\lambda_{ab} = 2230$ Å.

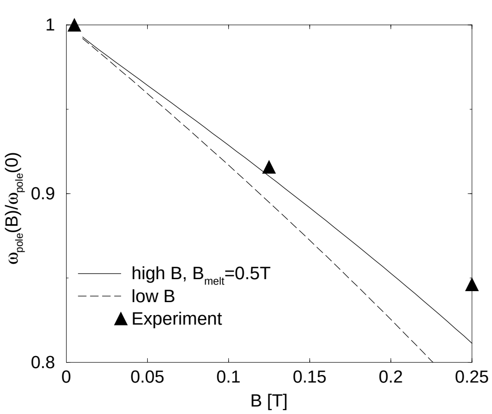

FIG. 4. Best fit of $\omega_{\rm pole}/\omega_{\rm pole}(0)$ for high temperature $T = 10$ K using the low and high field limits in Eq. (10) ($\lambda_{ab} = 1600$ Å) and Eq. (19) ($\lambda_{ab} = 1100$ Å, $B_{\rm melt} = 0.5$ T).

We now compare the calculated frequencies $\omega_m(B)$ with the experimental results in Ref. [6], where the eigenfrequencies $\omega_{0m}(B)$ are extracted from the zeros of the effective dielectric function $\epsilon_{\rm eff}$ and the characteristic frequency $\omega_{\rm pole}$ from the peak in $\mathrm{Im}(\omega\epsilon_{\rm eff})$.

Firstly, we use the normalized values of $\omega_{\rm pole}(B)/\omega_{\rm pole}(0)$, as they do not depend on the choice of the charge coupling parameter α, see Figs. 3 and 4. For low fields the results derived in the cases $B \ll B_\lambda, B_{Jm}$ (Eq. (10)) and $B \gg B_\lambda, B_{Jm}$ (Eq. (19)) are close, but deviate at fields approaching the melting field $B_{\rm melt} \approx 0.5$ T, which is chosen in accordance with the observed phase transition to the vortex liquid. The agreement with the experimental data is worsening near the vortex solid-liquid transition.

For low temperatures, cf. Fig. 3, agreement with the experimental data can be obtained for $\lambda_{ab} = 1850$ Å, while at high $T = 10$ K we get $\lambda_{ab} = 1600$ Å using Eq. (10) or $\lambda_{ab} = 1100$ Å ($B_m = 0.5$ T) from Eq. (19). Following from estimates for B_λ the latter case seems to be more adequate. For higher temperatures, cf. Fig. 4, the assumptions of our theory are expected to be better justified and in this case the obtained value for $\lambda_{ab} = 1100$ Å is comparable to the ones reported for other layered superconductors.

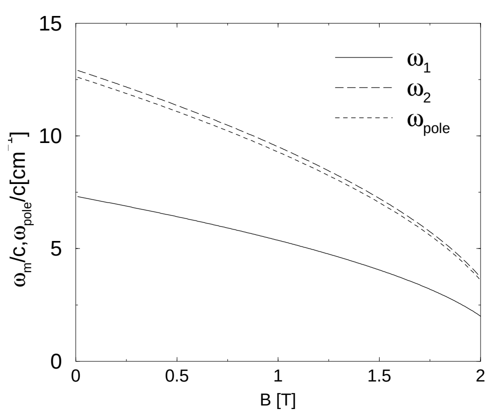

FIG. 5. JPR peaks $\omega_m(B)$ in $L(\omega)$ and $\omega_{\rm pole}(B)$ in $\mathrm{Im}(\omega\epsilon_{\rm eff})$ using the low field limit ($\lambda_{ab} = 1700$ Å, $T = 2.3$ K, $\alpha = 0.4$, $\omega_{c1}(B=0)/c = 6.6$ cm^{-1}, $\omega_{c2}(B=0)/c = 8.9$ cm^{-1}, cf. Ref. [6,15]).

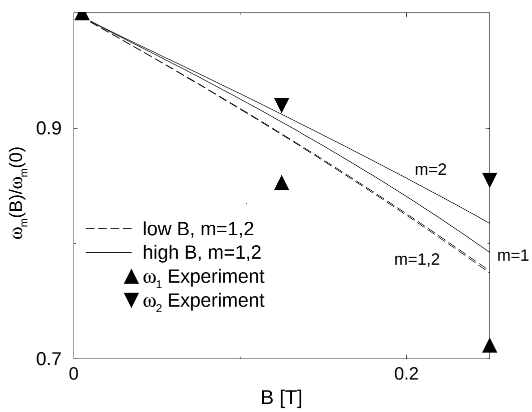

FIG. 6. Best fit of $\omega_{0m}(B)/\omega_{0m}(0)$ from Ref. [6] for high temperatures $T = 10$K using the low (dashed, $\lambda_{ab} = 1600$Å) and high (solid, $\lambda_{ab} = 1100$Å, $B_{\mathrm{melt}} = 0.5$T) field approximations, Eqs. (10) and (19) respectively ($\alpha = 0.4$).

Assuming the charge coupling $\alpha \approx 0.4$ as estimated in Ref. [12] from the vortex liquid state and the shape of the loss function for $B = 0$, the resonance frequencies $\omega_m(B, \alpha)$ in $L(\omega)$ and ω_{pole} are shown in Fig. 5. The experimental data for the normalized JPR resonances $\omega_m(B)/\omega_m(0)$ for high temperatures can be fitted with the same choice of parameters as for ω_{pole}, cf. Fig. 6.

To conclude, thermal fluctuations in the vortex solid state describe satisfactorily the dependence of the plasma frequencies in the Josephson coupled system $SmLa_{1-x}Sr_xCuO_{4-\delta}$ with two different alternating junctions on a perpendicular magnetic field. From the comparison of our theoretical results and the experimental data we estimate the in-plane London penetration length $\lambda_{ab} \approx 1100$ Å at high temperatures $T = 10$ K, which is similar to other layered superconductors.

The authors thank Dirk van der Marel and A. Pimenov for useful discussions and providing the experimental data. This work was supported by the Los Alamos National Laboratory under the auspices of the US DOE and by the Swiss National Science Foundation through the National Center of Competence in Research "Materials with Novel Electronic Properties-MaNEP".

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[15] In Ref. [6] the frequency ω/c is given in units of cm^{-1}. Therefore $\omega/c = 6.6$cm^{-1} and $\epsilon_0 = 19$ corresponds to the wavelength 3.5 10^6 Å. (A. Pimenov, private communication).